Name:____________________

Section 5.2
Right Triangle Trigonometry

Measuring Up, Way Up

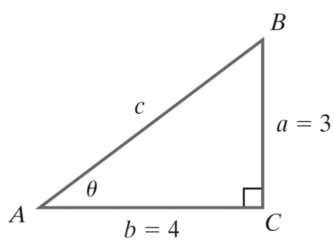
Did you ever wonder how you could measure the height of a building or a tree? How can you find the distance across a lake or some other body of water?

In this section, we show how to model such situations using a right triangle and then using relationships among the lengths its sides and the measures of its angles to find distances that are otherwise difficult to measure. These relationships are known as the trigonometric functions.

Objective #1: Use right triangles to evaluate trigonometric functions.

✔ *Solved Problem #1*

1a. Find the value of each of the six trigonometric functions of θ in the figure.



We first need to find c, the length of the hypotenuse. We use the Pythagorean Theorem.

$$c^2 = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$c = \sqrt{25} = 5$$

We apply the definitions of the six trigonometric functions. Note that the side labeled $a = 3$ is opposite angle θ and the side labeled $b = 4$ is adjacent to angle θ.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

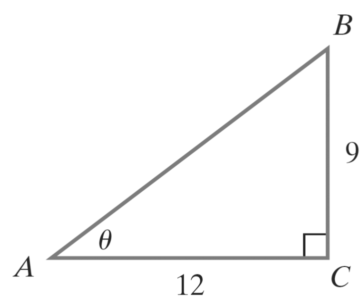
$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{3}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{4}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{3}$$

✎ *Pencil Problem #1* ✎

1a. Find the value of each of the six trigonometric functions of θ in the figure.



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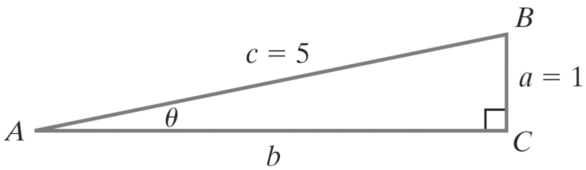
1b. Find the value of each of the six trigonometric functions of θ in the figure. Express each value in simplified form.



We first need to find b.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 1^2 + b^2 &= 5^2 \\ 1 + b^2 &= 25 \\ b^2 &= 24 \\ b &= \sqrt{24} = 2\sqrt{6} \end{aligned}$$

We apply the definitions of the six trigonometric functions.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2\sqrt{6}}{5}$$

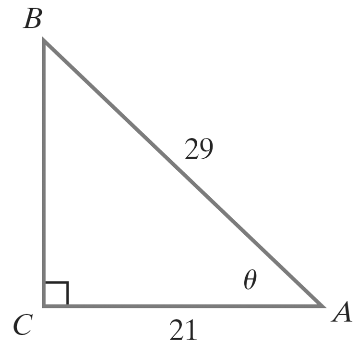
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{2\sqrt{6}} = \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{5}{1} = 5$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{5}{2\sqrt{6}} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{2\sqrt{6}}{1} = 2\sqrt{6}$$

1b. Find the value of each of the six trigonometric functions of θ in the figure. Express each value in simplified form.



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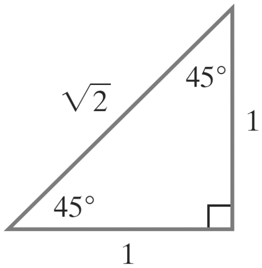
Objective #2: Find function values for $30°\left(\frac{\pi}{6}\right)$, $45°\left(\frac{\pi}{4}\right)$, and $60°\left(\frac{\pi}{3}\right)$.

✔ *Solved Problem #2*

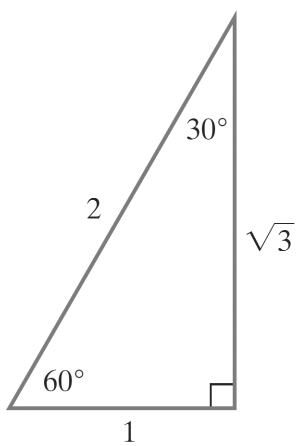
2a. Use the right triangle to find $\csc 45°$.



Use the definition of the cosecant function.

$$\csc 45° = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

2b. Use the right triangle to find $\tan 60°$.



Use the definition of the tangent function and the angle marked 60° in the triangle.

$$\tan 60° = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

✎ *Pencil Problem #2* ✎

2a. Use the right triangle in Solved Problem #2a to find $\sec 45°$.

2b. Use the right triangle in Solved Problem #2b to find $\cos 30°$.

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Objective #3: Recognize and use fundamental identities.

✔ *Solved Problem #3*

3a. Given $\sin\theta = \frac{2}{3}$ and $\cos\theta = \frac{\sqrt{5}}{3}$, find the value of each of the four remaining trigonometric functions.

Find $\tan\theta$ using a quotient identity.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{3}\cdot\frac{3}{\sqrt{5}} = \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Use reciprocal identities to find the remaining three function values.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2}{\sqrt{5}}} = \frac{\sqrt{5}}{2}$$

3b. Given that $\sin\theta = \frac{1}{2}$ and θ is an acute angle, find the value of $\cos\theta$ using a trigonometric identity.

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. Because θ is an acute angle, $\cos\theta$ is positive.

$$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\cos\theta = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

✎ *Pencil Problem #3* ✐

3a. Given $\sin\theta = \frac{1}{3}$ and $\cos\theta = \frac{2\sqrt{2}}{3}$, find the value of each of the four remaining trigonometric functions.

3b. Given that $\sin\theta = \frac{6}{7}$ and θ is an acute angle, find the value of $\cos\theta$ using a trigonometric identity.

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Objective #4: Use cofunctions of complements.

✔ *Solved Problem #4*

4a. Find a cofunction with the same value as $\sin 46°$.

$\sin 46° = \cos(90° - 46°) = \cos 44°$

4b. Find a cofunction with the same value as $\cot \frac{\pi}{12}$.

$$\cot \frac{\pi}{12} = \tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \tan\left(\frac{6\pi}{12} - \frac{\pi}{12}\right) = \tan\left(\frac{5\pi}{12}\right)$$

✎ *Pencil Problem #4* ✐

4a. Find a cofunction with the same value as $\sin 7°$.

4b. Find a cofunction with the same value as $\tan \frac{\pi}{9}$.

Objective #5: Evaluate trigonometric functions with a calculator.

✔ *Solved Problem #5*

5a. Use a calculator to find the value of $\sin 72.8°$ to four decimal places.

Use degree mode.

On a scientific calculator, enter the angle measure, 72.8, and then press the SIN key.

On a graphing calculator, press the SIN key, and then enter the angle measure, 72.8, and press ENTER.

The display, rounded to four places, should be 0.9553.

5b. Use a calculator to find the value of $\csc 1.5$ to four decimal places.

Use radian mode.

On a scientific calculator, enter the angle measure, 1.5, and then press the SIN key followed by the reciprocal key labeled $1/x$.

On a graphing calculator, open a set of parentheses, press the SIN key, and then enter the angle measure, 1.5. Close the parentheses, press the reciprocal key labeled x^{-1}, and press ENTER. The display, rounded to four places, should be 1.0025.

✎ *Pencil Problem #5* ✐

5a. Use a calculator to find the value of $\tan 32.7°$ to four decimal places.

5b. Use a calculator to find the value of $\cot \frac{\pi}{12}$ to four decimal places.

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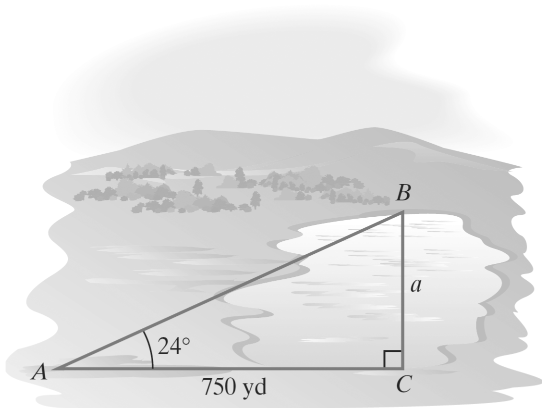
Objective #6: Use right triangle trigonometry to solve applied problems.

✔ Solved Problem #6

6. The distance across a lake, *a*, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?



We know the measurements of one angle and the leg adjacent to the angle. We need to know the length of the side opposite the known angle. We use the tangent function.

$$\tan 24° = \frac{a}{750}$$
$$a = 750 \tan 24°$$
$$a \approx 333.9$$

The distance across the lake is approximately 333.9 yards.

✎ Pencil Problem #6 ✎

6. To find the distance across a lake, a surveyor took the measurements shown in the figure. Use the measurements to determine how far it is across the lake. Round to the nearest yard.