Section 5.1
Angles and Radian Measure

Ever Feel Like You're Just Going in Circles?

You're riding on a Ferris wheel and wonder how fast you are traveling. Before you got on the ride, the operator told you that the wheel completes two full revolutions every minute and that your seat is 25 feet from the center of the wheel. You just rode on the merry-go-round, which made 2.5 complete revolutions per minute. Your wooden horse was 20 feet from the center, but your friend, riding beside you was only 15 feet from the center. Were you and your friend traveling at the same rate?
In this section, we study both angular speed and linear speed and solve problems similar to those just stated.

Objective #1: Recognize and use the vocabulary of angles.

✔ *Solved Problem #1*

1a. True or false: When an angle is in standard position, its initial side is along the positive y-axis.

False; When an angle is in standard position, its initial side is along the positive x-axis.

1b. Fill in the blank to make a true statement: If the terminal side of an angle in standard position lies on the x-axis or the y-axis, the angle is called a/an ___________ angle.

Such an angle is called a quadrantal angle.

✎ *Pencil Problem #1* ✐

1a. True or false: When an angle is in standard position, its vertex lies in quadrant I.

1b. Fill in the blank to make a true statement: A negative angle is generated by a __________ rotation.

Objective #2: Use degree measure.

✔ *Solved Problem #2*

2. Fill in the blank to make a true statement: An angle that is formed by $\frac{1}{2}$ of a complete rotation measures _________ degrees and is called a/an _________ angle.

Such an angle measures 180 degrees and is called a straight angle.

✎ *Pencil Problem #2* ✐

2. Fill in the blank to make a true statement: An angle that is formed by $\frac{1}{4}$ of a complete rotation measures _________ degrees and is called a/an _________ angle.

Objective #3: Use radian measure.

✔ *Solved Problem #3*

3. A central angle, θ, in a circle of radius 12 feet intercepts an arc of length 42 feet. What is the radian measure of θ?

The radian measure of the central angle, θ, is the length of the intercepted arc, s, divided by the radius of the circle, r: $\theta = \frac{s}{r}$. In this case, s = 42 feet and r = 12 feet.

$$\theta = \frac{s}{r} = \frac{42 \text{ feet}}{12 \text{ feet}} = 3.5$$

The radian measure of θ is 3.5.

✎ *Pencil Problem #3* ✐

3. A central angle, θ, in a circle of radius 10 inches intercepts an arc of length 40 inches. What is the radian measure of θ?

Objective #4: Convert between degrees and radians.

✔ *Solved Problem #4*

4a. Convert 60° to radians.
To convert from degrees to radians, multiply by $\frac{\pi \text{ radians}}{180°}$. Then simplify.

$$60° \cdot \frac{\pi \text{ radians}}{180°} = \frac{60\pi \text{ radians}}{180} = \frac{\pi}{3} \text{ radians}$$

4b. Convert −300° to radians.

$$-300° \cdot \frac{\pi \text{ radians}}{180°} = -\frac{300\pi \text{ radians}}{180} = -\frac{5\pi}{3} \text{ radians}$$

✎ *Pencil Problem #4* ✐

4a. Convert 135° to radians. Express your answer as a multiple of π.

4b. Convert −225° to radians. Express your answer as a multiple of π.

4c. Convert $\frac{\pi}{4}$ radians to degrees.

To convert from radians to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$. Then simplify.

$$\frac{\pi}{4} \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = \frac{180°}{4} = 45°$$

4c. Convert $\frac{\pi}{2}$ radians to degrees.

4d. Convert 6 radians to degrees.

$$6 \text{ radians} \cdot \frac{180°}{\pi \text{ radians}} = \frac{1080°}{\pi} \approx 343.8°$$

4d. Convert 2 radians to degrees. Round to two decimal places.

Objective #5: Draw angles in standard position.

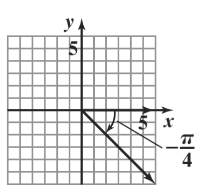
✔ *Solved Problem #5*

5a. Draw the angle $\theta = -\frac{\pi}{4}$ in standard position.

Since the angle is negative, it is obtained by a clockwise rotation. Express the angle as a fractional part of 2π.

$$\left|-\frac{\pi}{4}\right| = \frac{\pi}{4} = \frac{1}{8} \cdot 2\pi$$

The angle $\theta = -\frac{\pi}{4}$ is $\frac{1}{8}$ of a full rotation in the clockwise direction.



✎ *Pencil Problem #5* ✎

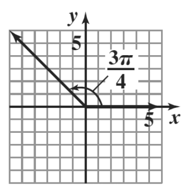
5a. Draw the angle $\theta = -\frac{5\pi}{4}$ in standard position.

5b. Draw the angle $\alpha = \frac{3\pi}{4}$ in standard position.

Since the angle is positive, it is obtained by a counterclockwise rotation. Express the angle as a fractional part of 2π.

$$\frac{3\pi}{4} = \frac{3}{8} \cdot 2\pi$$

The angle $\alpha = \frac{3\pi}{4}$ is $\frac{3}{8}$ of a full rotation in the counterclockwise direction.



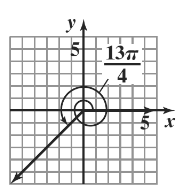
5b. Draw the angle $\alpha = \frac{7\pi}{6}$ in standard position.

5c. Draw the angle $\gamma = \frac{13\pi}{4}$ in standard position.

Since the angle is positive, it is obtained by a counterclockwise rotation. Express the angle as a fractional part of 2π.

$$\frac{13\pi}{4} = \frac{13}{8} \cdot 2\pi$$

The angle $\gamma = \frac{13\pi}{4}$ is $\frac{13}{8}$ or $1\frac{5}{8}$ full rotation in the counterclockwise direction. Complete one full rotation and then $\frac{5}{8}$ of a full rotation.



5c. Draw the angle $\gamma = \frac{16\pi}{3}$ in standard position.

Objective #6: Find coterminal angles.

✔ *Solved Problem #6*

6a. Find a positive angle less than 360° that is coterminal with a 400° angle.

Since 400° is greater than 360°, we subtract 360°.

$400° - 360° = 40°$

A 40° angle is positive, less than 360°, and coterminal with a 400° angle.

6b. Find a positive angle less than 2π that is coterminal with a $-\frac{\pi}{15}$ angle.

Since $-\frac{\pi}{15}$ is negative, we add 2π.

$$-\frac{\pi}{15}+2\pi=-\frac{\pi}{15}+\frac{30\pi}{15}=\frac{29\pi}{15}$$

A $\frac{29\pi}{15}$ angle is positive, less than 2π, and coterminal with a $-\frac{\pi}{15}$ angle.

✎ *Pencil Problem #6* ✐

6a. Find a positive angle less than 360° that is coterminal with a 395° angle.

6b. Find a positive angle less than 2π that is coterminal with a $-\frac{\pi}{50}$ angle.

6c. Find a positive angle less than 2π that is coterminal with a $\frac{17\pi}{3}$ angle.

Since $\frac{17\pi}{3}$ is greater than 4π, we subtract two multiples of 2π.

$$\frac{17\pi}{3} - 2 \cdot 2\pi = \frac{17\pi}{3} - 4\pi = \frac{17\pi}{3} - \frac{12\pi}{3} = \frac{5\pi}{3}$$

A $\frac{5\pi}{3}$ angle is positive, less than 2π, and coterminal with a $\frac{17\pi}{3}$ angle.

6c. Find a positive angle less than 2π that is coterminal with a $-\frac{31\pi}{7}$ angle.

Objective #7: Find the length of a circular arc.

✔ *Solved Problem #7*

7. A circle has a radius of 6 inches. Find the length of the arc intercepted by a central angle of 45°. Express arc length in terms of π. Then round your answer to two decimal places.

We begin by converting 45° to radians.

$$45^{\circ} \cdot \frac{\pi \text{ radians}}{180^{\circ}} = \frac{45\pi \text{ radians}}{180} = \frac{\pi}{4} \text{ radians}$$

Now we use the arc length formula $s = r\theta$ with the radius $r = 6$ inches and the angle $\theta = \frac{\pi}{4}$ radians.

$$s = r\theta = (6 \text{ in.})\left(\frac{\pi}{4}\right) = \frac{6\pi}{4} \text{ in.} = \frac{3\pi}{2} \text{ in.} \approx 4.71 \text{ in.}$$

✎ *Pencil Problem #7* ✐

7. A circle has a radius of 8 feet. Find the length of the arc intercepted by a central angle of 225°. Express arc length in terms of π. Then round your answer to two decimal places.

Objective #8: Use linear and angular speed to describe motion on a circular path.

✔ *Solved Problem #8*

8. A 45-rpm record has an angular speed of 45 revolutions per minute. Find the linear speed, in inches per minute, at the point where the needle is 1.5 inches from the record's center.

We are given the angular speed in revolutions per minute: $\omega = 45$ revolutions per minute. We must express ω in radians per minute.

$$\omega = \frac{45\ \text{revolutions}}{1\ \text{minute}} \cdot \frac{2\pi\ \text{radians}}{1\ \text{revolution}}$$

$$= \frac{90\pi\ \text{radians}}{1\ \text{minute}} \text{ or } \frac{90\pi}{1\ \text{minute}}$$

Now we use the formula $v = r\omega$.

$$v = r\omega = 1.5\ \text{in.} \cdot \frac{90\pi}{1\ \text{min}} = \frac{135\pi\ \text{in.}}{\text{min}} \approx 424\ \text{in./min}$$

✎ *Pencil Problem #8* ✐

8. A Ferris wheel has a radius of 25 feet. The wheel is rotating at two revolutions per minute. Find the linear speed, in feet per minute, of a seat on this Ferris wheel.