

The Binomial Theorem

Lesson Notes

$$t_{k+1} = {}_n C_k (x)^{n-k} (y)^k$$

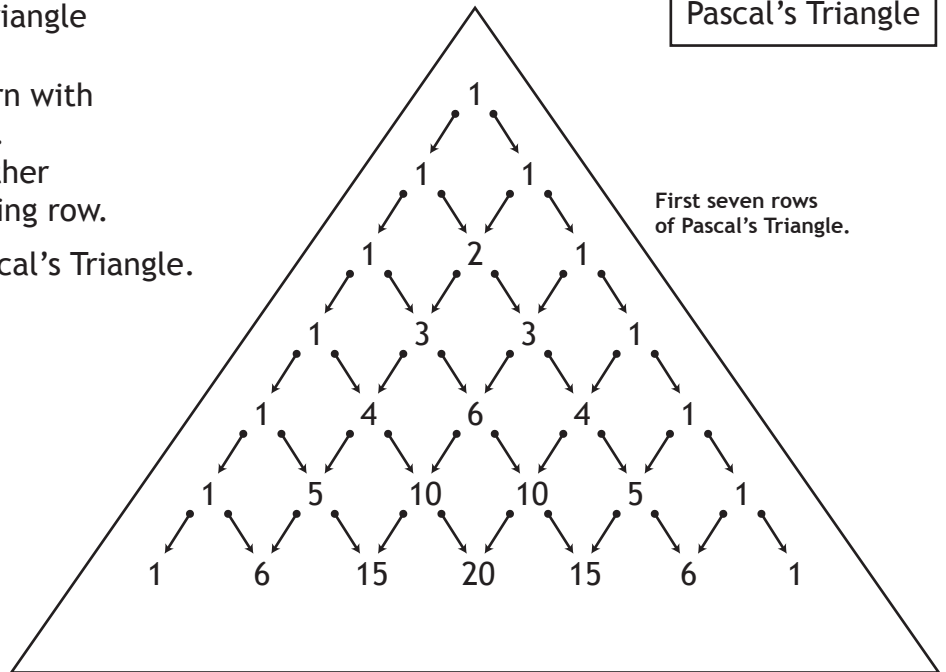
Example 1

Pascal's Triangle

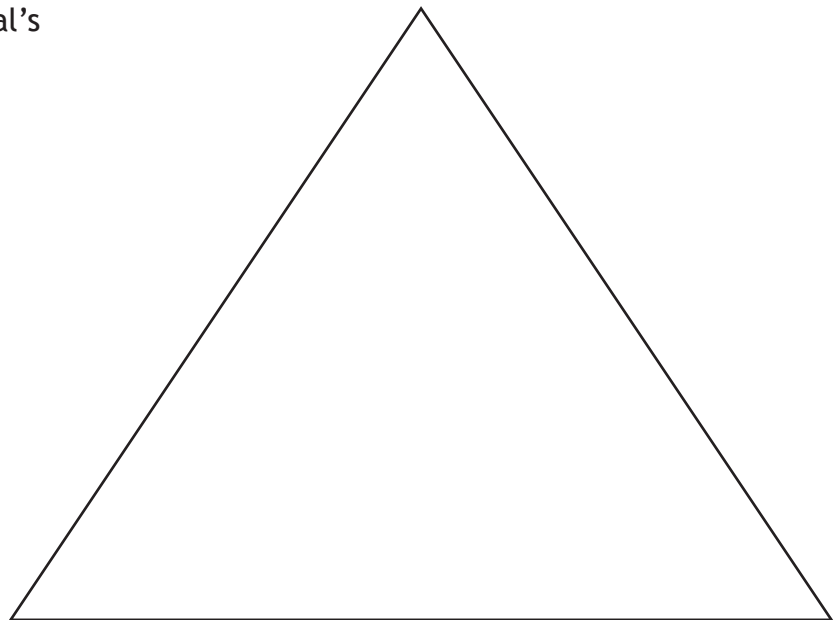
Pascal's Triangle

Pascal's Triangle is a number pattern with useful applications in mathematics. Each row is formed by adding together adjacent numbers from the preceding row.

a) Determine the eighth row of Pascal's Triangle.



b) Rewrite the first seven rows of Pascal's Triangle, but use combination notation instead of numbers.



c) Using the triangles from parts (a & b) as a reference, explain what is meant by ${}_n C_k = {}_n C_{n-k}$.

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Example 2

Rows and Terms of Pascal's Triangle.

Pascal's Triangle

a) Given the following rows from Pascal's Triangle, write the circled number as a combination.

i) $\textcircled{1}$ 8 28 56 70 56 28 8 1

ii) 1 12 66 220 495 792 924 792 495 220 $\textcircled{66}$ 12 1

b) Use a combination to find the third term in row 22 of Pascal's Triangle.

c) Which positions in the 12th row of Pascal's Triangle have a value of 165?

d) Find the sum of the numbers in each of the first four rows of Pascal's Triangle. Use your result to derive a function, $S(n)$, for the sum of all numbers in the n^{th} row of Pascal's Triangle. What is the sum of all numbers in the eleventh row?

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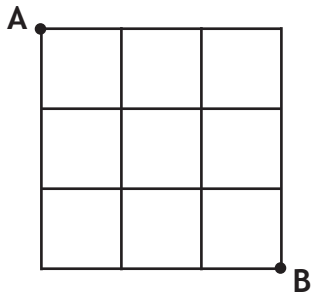
$$t_{k+1} = {}_n C_k (x)^{n-k} (y)^k$$

Example 3

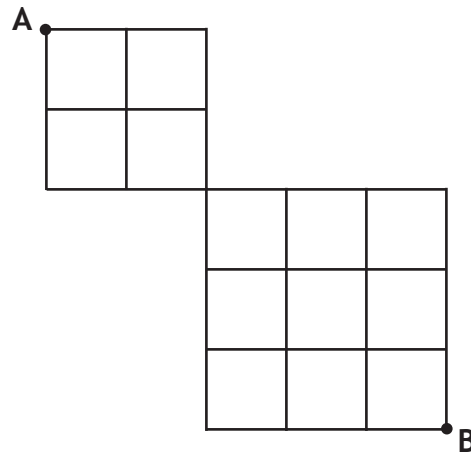
Use Pascal's Triangle to determine the number of paths from point A to point B if east and south are the only possible directions.

Pascal's Triangle and Pathways

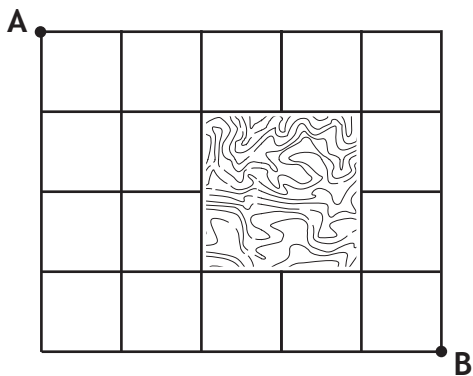
a)



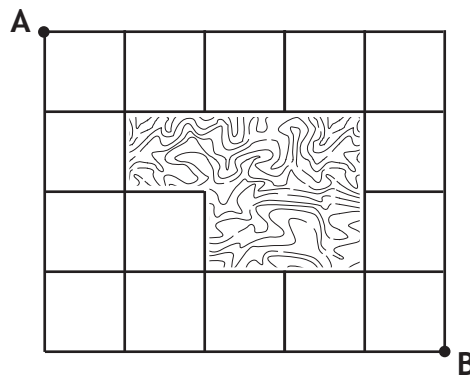
b)



c)



d)



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Example 4

The Binomial Theorem.

The Binomial Theorem

a) Define the binomial theorem and explain how it is used to expand $(x + 1)^3$.

Expand the expressions in parts (b) and (c) using the binomial theorem.

b) $(x + 2)^6$

c) $(2x - 3)^4$

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Example 5

Expand each expression.

The Binomial Theorem

a) $(x^2 - 2y)^4$

b) $\left(3x^2 - \frac{1}{2}\right)^4$

c) $\left(2x^3 - \frac{3}{x}\right)^5$

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Example 6

Write each expression as a binomial power.

Condense
to a Binomial

a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

b) $32a^5 - 240a^4b + 720a^3b^2 - 1080a^2b^3 + 810ab^4 - 243b^5$

c) $27a^3 - \frac{27a^2}{4} + \frac{9a}{16} - \frac{1}{64}$

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Example 7

Use the general term formula to find the requested term in a binomial expansion.

General Term

$$t_{k+1} = {}_n C_k (x)^{n-k} (y)^k$$

a) Find the third term in the expansion of $(x - 3)^4$.

b) Find the fifth term in the expansion of $(3a^3 - 2b^2)^8$.

c) Find the fourth term in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$.

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Example 8

Answer each of the following questions.

Finding Specific Values

a) In the expansion of $(5a - 2b)^9$, what is the coefficient of the term containing a^5 ?

b) In the expansion of $(4a^3 + 3b^3)^5$, what is the coefficient of the term containing b^{12} ?

c) In the expansion of $(3a - 4)^8$, what is the middle term?

d) If there are 23 terms in the expansion of $(a - 2)^{3k-5}$, what is the value of k ?