

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

Combinations Lesson Notes

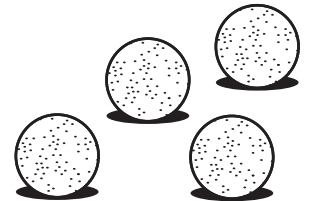
Example 1

Introduction to Combinations.

Combinations

There are four marbles on a table, and each marble is a different color (red, green, blue, and yellow). Two marbles are selected from the table at random and put in a bag.

a) Is the order of the marbles, or the order of their colors, important?



b) Use a tree diagram to find the number of unique color combinations for the two marbles.

c) Use combination notation to find the number of unique color combinations.

d) What is meant by the terms *single-case combination* and *multi-case combination*?

e) How many ways can three or four marbles be chosen?

Combinations Lesson Notes

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Example 2

Combinations with
Repetitions NOT Allowed.
(Finite Sample Sets)

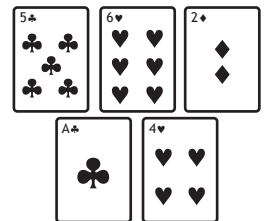
Sample Sets with NO Subdivisions

Single-Case
Combinations

a) There are five toppings available for a pizza (*mushrooms, onions, pineapple, spinach, and tomatoes*). If a pizza is ordered with three toppings, and no topping may be repeated, how many different pizzas can be created?

b) A committee of 4 people is to be formed from a selection pool of 9 people. How many possible committees can be formed?

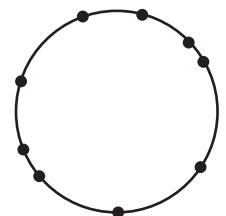
c) How many 5-card hands can be made from a standard deck of 52 cards?



d) There are 9 dots randomly placed on a circle.

i) How many lines can be formed within the circle by connecting two dots?

ii) How many triangles can be formed within the circle?



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Combinations Lesson Notes

Example 3

Combinations with
Repetitions NOT Allowed.
(*Finite Sample Sets*)

Sample Sets with Subdivisions

Single-Case
Combinations

a) How many 6-person committees can be formed from 11 men and 9 women if 3 men and 3 women must be on the committee?

b) A crate of toy cars contains 10 working cars and 4 defective cars. How many ways can 5 cars be selected if only 3 work?

c) From a deck of 52 cards, a 6-card hand is dealt. How many distinct hands are there if the hand must contain 2 spades and 3 diamonds?

d) A bouquet contains four types of flowers:

Flower Type	Examples
Focal Flowers: Large and eye-catching flowers that draw attention to one area of the bouquet.	Roses, Peonies, Hydrangeas, Chrysanthemums, Tulips, and Lilies
Fragrant Flowers: Flowers that add a pleasant fragrance to the bouquet.	Petunia, Daffodils, Daphnes, Gardenia, Lilacs, Violets, Magnolias
Line Flowers: Tall and narrow flowers used to establish the height of a floral bouquet.	Delphiniums, Snapdragons, Bells of Ireland, Gladioli, and Liatris
Filler Flowers: Unobtrusive flowers that give depth to the bouquet.	Daisies, Baby's Breath, Wax Flowers, Solidago, and Caspia

A florist is making a bouquet that uses one type of focal flower, no fragrant flowers, three types of line flowers and all of the filler flowers. How many different bouquets can be made?

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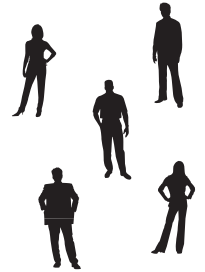
Example 4

Combinations with
Repetitions NOT Allowed.
(*Finite Sample Sets*)

More Sample Sets
with Subdivisions

Single-Case
Combinations

a) A committee of 5 people is to be formed from a selection pool of 12 people. If Carmen must be on the committee, how many unique committees can be formed?



b) A committee of 6 people is to be formed from a selection pool of 11 people. If Grant and Helen must be on the committee, but Aaron must not be on the committee, how many unique committees can be formed?

c) Nine students are split into three equal-sized groups to work on a collaborative assignment. How many ways can this be done? Does the sample set need to be subdivided in this question?

d) From a deck of 52 cards, a 5-card hand is dealt. How many distinct 5-card hands are there if the ace of spades and two of diamonds must be in the hand?

e) A lottery ticket has 6 numbers from 1-49. Duplicate numbers are not allowed, and the order of the numbers does not matter. How many different lottery tickets contain the numbers 12, 24 and 48, but exclude the numbers 30 and 40?

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Combinations Lesson Notes

Example 5

Combinations with
Repetitions NOT Allowed.
(*Finite Sample Sets*)

Permutations and
Combinations Together

Single-Case
Combinations

a) How many five-letter words using letters from TRIANGLE can be made if the five-letter word must have two vowels and three consonants?

b) There are 4 men and 5 women on a committee selection pool. A three-person committee consisting of President, Vice-President, and Treasurer is being formed. How many ways can exactly two men be on the committee?

c) A music teacher is organizing a concert for her students. If there are six piano students and seven violin students, how many different concert programs are possible if four piano students and three violin students perform in an alternating arrangement?

Combinations

Lesson Notes

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

Example 6

Combinations with
Repetitions NOT Allowed.
(Finite Sample Sets)

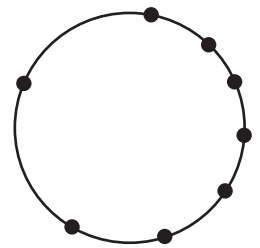
Handshakes, Teams,
and Shapes.

Single-Case
Combinations

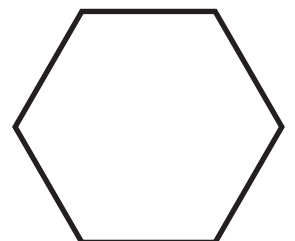
a) Twelve people at a party shake hands once with everyone else in the room. How many handshakes took place?

b) If each of the 8 teams in a league must play each other three times, how many games will be played? (Note: This is a multi-case combination)

c) If there are 8 dots on a circle, how many quadrilaterals can be formed?



d) A polygon has 6 sides. How many diagonals can be formed?



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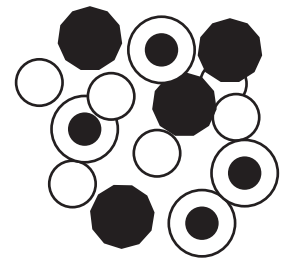
Combinations Lesson Notes

Example 7

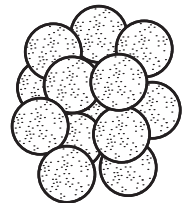
Combinations where Repetitions ARE Allowed.
(Infinite Sample Sets)

Single-Case
Combinations

a) A jar contains quarters, loonies, and toonies. If four coins are selected from the jar, how many unique coin combinations are there?



b) A bag contains marbles with four different colors (red, green, blue, and yellow). If three marbles are selected from the bag, how many unique color combinations are there?



Combinations Lesson Notes

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Example 8

More Than One Case (At Least/At Most).

Multi-Case
Combinations

a) A committee of 5 people is to be formed from a group of 4 men and 5 women. How many committees can be formed if at least 3 women are on the committee?

b) From a deck of 52 cards, a 5-card hand is dealt. How many distinct hands can be formed if there are at most 2 queens?

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Combinations Lesson Notes

c) From a deck of 52 cards, a 5-card hand is dealt. How many distinct hands can be formed if there is at least 1 red card?

d) A research team of 5 people is to be formed from 3 biologists, 5 chemists, 4 engineers, and 2 programmers. How many teams have exactly one chemist and at least 2 engineers?

e) In how many ways can you choose one or more of 5 different candies?

Combinations

Lesson Notes

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Example 9

Combination Formula.

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

a) Evaluate ${}_7 C_5$

b) Evaluate ${}_3 C_3$

c) Evaluate $\binom{4}{2}$

d) Write $\frac{6!}{4!2!}$ as a combination.

e) Write $\frac{5!}{4!}$ as a combination.