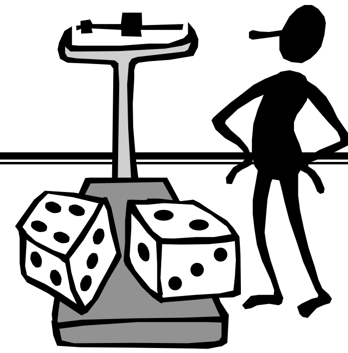
# Chapter 17: Probability Models



**Key Vocabulary:**

- Bernoulli trials
- Geometric model
- Binomial model

**Calculator Skills:**

- geometpdf(
- geometcdf(
- binompdf(
- binomcdf(

**Bernoulli Trials:**

A situation is called a ***Bernoulli trial*** if it meets the following criteria:

- There are only two possible outcomes (categorized as ________________ or ________________) for each trial
- The probability of success, denoted ______, is the same for each trial
- The trials are __________________________________

(Note that the independence assumption is violated whenever we sample without replacement, but is overridden by the __________ condition. As long as we don't sample more than 10% of the population, the probabilities don't change enough to matter.)

1. A new sales gimmick has 30% of the M&M's covered with speckles. These "groovy" candies are mixed randomly with the normal candies as they are put into the bags for distribution and sale. You buy a bag and remove candies one at a time looking for the speckles.

   a. Is this situation a Bernoulli trial? Explain.

   ______________________________________________________________________________

   ______________________________________________________________________________

b. What's the probability that the first speckled candy is the fourth one we draw from the bag?

____________________________________________________________________________

c. What's the probability that the first speckled candy is the tenth one?

____________________________________________________________________________

d. Write a general formula.

____________________________________________________________________________

e. What's the probability we find the first speckled one among the first three we look at?

____________________________________________________________________________

f. How many do we expect to have to check, on average, to find a speckled one?

____________________________________________________________________________

**Geometric Distributions:**

Suppose the random variable *X* = the number of trials required to obtain the first success. Then *X* is a

___________________ ___________________ ___________________ if:

1. There are only two outcomes: ___________________ or ___________________.
2. The probability of success *p* is ___________________ for each observation.
3. The *n* observations are ___________________.
4. The variable of interest is the ___________________________________________________.

Because *n* is not fixed there could be an infinite number of X values. However, the probability that *X* is a very large number is more and more unlikely. Therefore the probability histogram for a geometric distribution is always ______________________________________.

If *X* is a geometric random variable, it is said to have a ______________________________________, and is denoted as _______________.

The expected value (mean) of a geometric random variable is ______________.

The standard deviation of a geometric random variable is ___________________.

The probability that X is equal to *x* is given by the following formula:

_________________________________________________________________________________

2. Refer back to the M&M's distribution in problem 1.

   a. What's the probability that we'll find two speckled ones in a handful of five candies?

      ____________________________________________________________________________

   b. List all possible combinations of exactly two speckled M&M's in a handful of five candies.

      ____________________________________________________________________________

**Binomial Distributions:**

Suppose the random variable *X* = the number of successes in *n* observations.

Then *X* is a ____________________ ____________________ ____________________ if:

1. There are only two outcomes: ____________________ or ____________________.
2. The probability of success *p* is ____________________ for each observation.
3. The *n* observations are ____________________.
4. There is a _______________________________________ *n* of observations.

If *X* is a binomial random variable, it is said to have a _______________________________________,

and is denoted as _______________.

The expected value (mean) of a binomial random variable is ______________.

The standard deviation of a binomial random variable is ___________________.

The ____________________ ____________________ ____________________ (or ___________) assigns a probability to each value of *X*.

The ____________________ ____________________ ____________________ (or____________) calculates the sum of the probabilities up to *X*.

*Example*: Suppose each child born to Jay and Kay has probability 0.25 of having blood type O. If Jay and Kay have 5 children, what is the probability that exactly 2 of them have type O blood?

Let X = _______________________________________________________________________________.

1. There are only two outcomes: success (_______________) or failure (_______________).
2. The probability of success (_______________) is _______________ for each of the ________ observations.
3. Each of the 5 observations is ____________________, since one child's blood type will not influence the next child's blood type.
4. There is a fixed number of observations: __________.

So X is a ____________________________________________________________.

The following table shows the probability distribution function (__________) for the binomial random variable, X.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 0.2373 | 0.3955 | 0.2637 | 0.0879 | 0.0146 | 0.001 |

P(X = 0) = P( ) = ( $)^5$ = 0.2373

Binompdf ( ) = 0.2373
Binompdf ( ) = 0.3955
Binompdf ( ) = 0.2637
Binompdf ( ) = 0.0879
Binompdf ( ) = 0.0146
Binompdf ( ) = 0.0010

**Problem:** Construct a histogram of the p.d.f. using the window $X_1$ [0, 6] and $Y_{0.1}$ [0, 1].

The following table shows the cumulative distribution function (__________) for the binomial random variable, X.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(X=x)$ | 0.2373 | 0.3955 | 0.2637 | 0.0879 | 0.0146 | 0.001 |
| $P(X \leq x)$ | | | | | | |

Binomcdf ( ) = 0.2373
Binomcdf ( ) = 0.6328
Binomcdf ( ) = 0. 8965
Binomcdf ( ) = 0. 9844
Binomcdf ( ) = 0. 999
Binomcdf ( ) = 1

**Problem:** Construct a histogram of the c.d.f. using the window $X_1$ [0, 6] and $Y_{0.1}$ [0, 1].

1. Suppose I have a group of 4 students and I want to choose 1 of them as a volunteer. In how many ways can I choose 1 out of 4 students?

**Call this "_____________________." There are __________ ways.**

2. Suppose I have a group of 4 students and I want to choose 2 of them as volunteers. In how many ways can I choose 2 out of 4 students?

**Call this "____________________." There are __________ ways.**

3. Suppose I have a group of 5 students and I want to choose 1 of them as a volunteer. In how many ways can I choose 1 out of 5 students?

**Call this "____________________." There are __________ ways.**

4. Suppose I have a group of 5 students and I want to choose 3 of them as a volunteer. In how many ways can I choose 3 out of 5 students?

**Call this "____________________." There are __________ ways.**

5. Suppose I have a group of 20 students and I want to choose 4 of them as a volunteer. In how many ways can I choose 4 out of 20 students?

SSSSFFFFFFFFFFFFFFFF
FSSSSFFFFFFFFFFFFFFF
FSFFFFSFFFSFFFFFFFSF
… and so on…

**Call this "____________________." There are __________ ways.**

There is a mathematical way to count the total number of ways to arrange *k* out of *n* objects. This is

called **"**_________________**"** or the ____________________________________________.

The binomial coefficient is the number of ways to arrange *k* successes in *n* observations.

It is written __________ and is called "___________________."

The value of "*n* choose *k*" is given by the formula:

_____________________________________________________________________________________

Example: "5 choose 2"

________________________________________________________________________________

________________________________________________________________________________

So there are __________ ways to arrange 2 out of 5 objects.

Think of this as flipping a coin 5 times and getting 2 heads. In how many ways can that happen? It can happen in __________ ways.

If X is a binomial random variable with parameters *n* and *p*, then

$P\left(X = x\right) =$ ________________________________

# Binomial Experiments
## Examples/Problems

Binomial Experiments: A binomial experiment is one in which there are only two possible outcomes: success or failure. Also, the probability must remain constant throughout the experiment.

**Example 1:** Is flipping a coin a binomial experiment?
A coin being tossed will result in only two outcomes, heads or tails. The probability will remain 0.5 throughout the experiment since the probability of a head is ½, and the probability of a tail is ½ . Because of this, it is a binomial experiment.

**Example 2:** 4 marbles are in a box. If two marbles are drawn out successively without replacement, is this a binomial experiment?
This is ***not*** a binomial experiment since the probability for the second marble has changed. (We've changed the situation from pulling one marble out of four to pulling one marble out of three.)

## Questions:

For each of the following, determine if it is a binomial experiment or not.

1) A coin is flipped 20 times. What is the probability of getting 8 tails?

2) 60% of students from a particular high school take a physics course. What is the probability that out of 15 students from this school, 5 have taken a physics course?

3) The probability of pulling a diamond out of a standard deck of 52 cards is 13/52. What is the probability of pulling 2 diamonds out consecutively, without replacement?

4) The probability of pulling a diamond out of a standard deck of 52 cards is 13/52. What is the probability of pulling 2 diamonds out consecutively, with replacement?

5) A student randomly guesses on seven multiple choice questions. If each question has four choices, find the probability the student gets 3 questions correct.

6) In a group of 10 laser printers, only 7 work. If a sample of 3 printers is taken, what is the probability that exactly 2 work?

7) A company knows that at any time in its production process, 95% of its laser printers work. What is the probability, to the nearest hundredth, that in a sample of 4 printers, exactly 3 work?

# Binomial Distributions
## Examples

**Binomial Distribution:** The following formula can be used to find the probability of success in a binomial experiment.

$$P(k) = {}_n C_k \, p^k (1-p)^{n-k}$$

***n*** = Total number of trials.
***p*** = Probability of success
***k*** = Number of successes
***P(k)*** = Probability of getting *k* successes.

**Example 1:** A coin is flipped 4 times. Determine the probability of obtaining no heads, one head, two heads, three heads, and four heads. Draw a probability histogram displaying your results.

The number of trials (*n*) is 4

The probability (*p*) of obtaining a head (which we will define as a success) is 0.5

We must do the calculation five times, once for each different value of *k*.

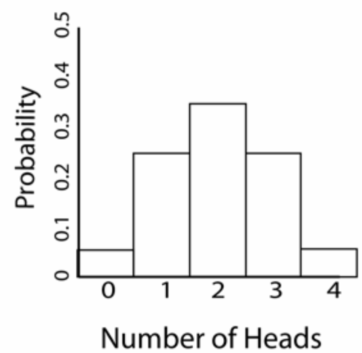
**Zero heads *(k=0)*** $P(0) = {}_4 C_0 (0.5)^0 (1-0.5)^{4-0} = 0.0625$

**One Head *(k=1)*** $P(1) = {}_4 C_1 (0.5)^1 (1-0.5)^{4-1} = 0.25$

**Two Heads *(k=2)*** $P(2) = {}_4 C_2 (0.5)^2 (1-0.5)^{4-2} = 0.375$

**Three Heads *(k=3)*** $P(3) = {}_4 C_3 (0.5)^3 (1-0.5)^{4-3} = 0.25$

**Four Heads *(k=4)*** $P(4) = {}_4 C_4 (0.5)^4 (1-0.5)^{4-4} = 0.0625$




**<u>*There is a way to do this calculation in your calculator very quickly!</u>**

Use the command binompdf (n, p, k) to evaluate a binomial probability in your calculator.
Find this by typing: 2nd → Distr → Binompdf. (scroll down to find this command, or type 0)

If you want to use this command to find only one case (e.g. *k* = 2) type in:
**binompdf(4 , 0.5 , 2)**

If you want to use this command to find all the cases, (k = 0 to 5) type in:
**binompdf(4 , 0.5 , {0 , 1, 2, 3, 4})** *(Make sure you use the proper brackets.)*
The answers for all five cases will be displayed, separated by commas. Scroll right to see them all.

If you want to find the sum of all the probabilities, you can type in:
**2nd → list → math → sum→Enter→2nd → ans**
Notice that the sum adds up to 1, since the above probabilities account for all possible cases!

# Binomial Distributions

## Examples

**Example 2:** A particular brand of CD player has a 20% chance of having a defect when it leaves the factory. If a store sells 7 of these CD players, what is the probability at most 2 have a defect?

The number of trials is 7
The probability of having a defect (*which in this questions counts as a success*) is 0.2

At most 2 players having a defect means we want to add up the probabilities of zero having a defect, one having a defect, and two having a defect.

Evaluate using **binompdf(7, 0.2, {0, 1, 2})**, then **sum(ans)**

**Answer = 0.852**

**Example 3:** 35% of university students regularly take the bus to school. If 13 students are randomly sampled, what is the probability at least 4 take the bus?

The number of trials is 13
The probability of taking the bus (which in this questions counts as a success) is 0.35

If we want the probability of at least 4 taking the bus, we could add the cases where $k$ = 4, 5, 6, … 13.
**binompdf(13, 0.35, {4,5,6,7,8,9,10,11,12,13})**, then **sum(ans) = 0.7217**

*Alternatively, you could subtract all the probabilities you **don't** want from 1.*

*Evaluate the unwanted cases using **binompdf(13, 0.35, {0, 1, 2, 3})**, then **sum(ans)** = 0.2783*

*Probability that 0 to 3 students take the bus = 0.2783*

*The probability at least 4 take the bus is 1 − 0.2783 = 0.7217*

**Example 4:** 3% of candies in a mix bag are peppermint. What is the probability that in a sample of 17 candies, at least 1 is peppermint?

The probability that no candy is peppermint is: binompdf(17, 0.03, 0) = 0.5958
The probability that at least one candy is peppermint is: 1 – 0.5958 = **0.4042**

# Binomial Distributions
## Problems

### Questions:

**1)** The probability that a high school student will graduate is 0.81. Out of 9 students, determine the probability that:

a) three students graduate.


b) 2 students do not graduate.

c) at least 7 students graduate.

d) at most 1 student graduates.

e) at least one student graduates.

**2)** The probability of a dart player getting a Bullseye is 0.04. Determine the probability that in five throws she will:

a) Score 1 Bullseye.

b) Score at most 1 Bullseye

c) Score at least 3 Bullseyes

3) What is the probability of drawing 2 red cards consecutively (*without replacement* ) from a standard deck of 52 cards?

**Assignment:** Chapter 17 Exercises Pg. 401 - 404; #9 - 33 odd